The Keçeci-Fujii Model Derived from Akdeniz-Dane Model:
2n-Dimensional Instanton Solutions, Higher-Derivative Interactions and Quantum Information Applications

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Abstract

This study presents the **Keçeci-Fujii model**, derived from the two-dimensional conformal model of Akdeniz and Dane and built upon Fujii's higher-derivative generalization. The model is a higher-derivative theory with scalar-spinor interaction in even-dimensional spaces. Recurrence relations for the Keçeci-Fujii model have been derived, relationships between coupling constants have been analyzed, and the comparative behavior of Gürsey, Akdeniz-Dane, and Keçeci-Fujii coupling constants in four-dimensional conformal models has been examined. The work discusses the model's potential applications in quantum computing, topological quantum materials, and holographic principles, and suggests future research directions.

Keywords: Instanton, Keçeci-Fujii model, Akdeniz-Dane model, Liouville term, conformal field theory, coupling constant, higher-derivative theories, quantum computing.

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PhySH: Conformal field theory; Solitons and instantons; Supersymmetry; Lagrangian approach; Quantum simulation; Topological phases of matter.

1 Introduction

The study of instanton-like solutions in conformal field theories [20] is of fundamental importance for understanding non-perturbative quantum phenomena. In this direction, the two-dimensional scalar-spinor interaction model proposed by Akdeniz and Dane [1] exhibits a rich structure arising from the combination of conformal symmetry and Liouville term.

Fujii [3] generalized this model to even-dimensional spaces by adding higher derivatives and demonstrated the existence of instanton-like solutions. In our previous work [4], we performed a deep analysis of the mathematical structure of the Fujii model, deriving **recurrence relations** and establishing consistency conditions between coupling constants.

In this study, we present this generalized structure, which we name the **Keçeci-Fujii model**, and discuss its significance in the context of modern physics. The historical development of the model can be summarized as follows:

- **(i) Akdeniz-Dane Model (1985)**: Two-dimensional conformal scalar-spinor interaction
- **(ii) Fujii Generalization (1989)**: Higher derivatives in even-dimensional spaces
- **(iii) Keçeci Analysis (2011)**: Recurrence relations and coupling constant analysis
- **(iv) Keçeci-Fujii (Keçeci-Akdeniz-Fujii) Model** : Modern applications and quantum information connections

This developmental process enables us to bridge classical conformal field theory with modern quantum technologies.

The role of instantons in supersymmetric and non-supersymmetric gauge theories has illuminated fundamental topics such as vacuum structure, chiral symmetry breaking, and θ-vacuum [7, 6]. Particularly, the suppression of instanton contributions in the presence of massless fermions is critically important for preserving vacuum energy in supersymmetric theories [8].

In recent years, deep connections have been discovered between conformal field theories (CFT) and quantum information theory. The AdS/CFT correspondence, holographic principle, and entropy/entanglement relationships offer new perspectives on how non-perturbative structures might emerge in quantum computing and condensed matter systems [10]. In this context, instanton-like solutions in higher-derivative models are being re-evaluated as potential platforms for topological quantum computation and quantum simulation.

2 Mathematical Formulation of the Keçeci-Fujii Model

2.1 Historical Origin: Akdeniz-Dane Model

The roots of our model lie in the two-dimensional conformal model of Akdeniz and Dane [1]:

$$\mathcal{L}_{\mathrm{AD}} = \frac{1}{2}(\partial_\mu \varphi)^2 + \frac{i}{2}\bar{\psi}\partial\!\!\!/\psi + \frac{m^2}{\beta^2}e^{\beta\varphi} + \frac{m}{2\sqrt{2}}e^{\beta\varphi/2}\bar{\psi}\psi + g(\bar{\psi}\psi)^2. \tag{1}$$

This model represents one of the simplest scalar-spinor interaction theories containing the Liouville term.

2.2 Fujii Generalization

Fujii [3] generalized this model to even-dimensional spaces ($\mathbb{R}^D$, $D = 2n$) by adding higher derivatives:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{F}} = & \frac{1}{2}(\partial_{\mu_{D/2}}\partial_{\mu_{(D-2)/2}}\cdots\partial_{\mu_1}\varphi)^2 \\
& + \frac{1}{2}(\partial_{\mu_{(D-2)/2}}\cdots\partial_{\mu_1}\bar{\psi})(i\partial\!\!\!/)(\partial_{\mu_{(D-2)/2}}\cdots\partial_{\mu_1}\psi) \\
& + \sum_{j=0}^{D}\alpha_j e^{(D-j)\beta\varphi/D}(\bar{\psi}\psi)^j.
\end{aligned} \tag{2}
$$

2.3 Keçeci-Fujii Model: Completed Form with Recurrence Relations

Using the recurrence relations we derived in our previous work [4], we express the **Keçeci-Fujii Lagrangian** as follows:

$$
\begin{aligned}
\boxed{\mathcal{L}_{\mathrm{KF}} =} & \boxed{\frac{1}{2}(\partial_{\mu_{D/2}}\partial_{\mu_{(D-2)/2}}\cdots\partial_{\mu_1}\varphi)^2} \\
& \boxed{+\frac{1}{2}(\partial_{\mu_{(D-2)/2}}\cdots\partial_{\mu_1}\bar{\psi})(i\partial\!\!\!/)(\partial_{\mu_{(D-2)/2}}\cdots\partial_{\mu_1}\psi)} \\
& \boxed{+\sum_{j=0}^{D}\alpha_j^{(\mathrm{KF})} e^{(D-j)\beta\varphi/D}(\bar{\psi}\psi)^j}
\end{aligned} \tag{3}
$$

where the coupling constants $\alpha_j^{(\mathrm{KF})}$ satisfy the following **Keçeci recurrence relations**:

$$
(\bar{C}C)^j = \mp\frac{(2A)^{j+1}(D-1)!}{2AD\alpha_{j+1}^{(\mathrm{KF})}(j+1)}, \quad j = 1,\ldots,D \tag{4}
$$

$$
\alpha_j^{(\mathrm{KF})} = \frac{D^2(j+1)\alpha_{j+1}^{(\mathrm{KF})}}{A\beta^2(D-j)}, \quad j = 0,\ldots,(D-1). \tag{5}
$$

2.4 Instanton-Like Solutions

The conformal instanton-like solutions of the Keçeci-Fujii model are:

$$
\varphi_{\mathrm{KF}} = \frac{1}{\beta}\log\left(\frac{2A}{1+X^2}\right)^D, \tag{6}
$$

$$
\psi_{\mathrm{KF}} = \frac{1}{1+X^2}(1+i\gamma\cdot X)C. \tag{7}
$$

These solutions satisfy the following **Keçeci-Fujii consistency equations**:

$$
\frac{D}{2}2^D(D-1)! + \beta\sum_{j=0}^{D-1}\alpha_j^{(\mathrm{KF})}\frac{D-j}{D}(2A)^{D-j}(\bar{C}C)^j = 0, \tag{8}
$$

$$
-2^D(D-1)! + \beta\sum_{j=1}^{D}\alpha_j^{(\mathrm{KF})}j(2A)^{D-j}(\bar{C}C)^{j-1} = 0. \tag{9}
$$

3 Original Contributions of the Model and Keçeci Analysis

3.1 Keçeci Recurrence Relations

The fundamental contribution of our previous work [4] was the derivation of recurrence relations that constrain the model's parameter space. These relations:

- Ensure consistency among the coupling constants $\alpha_j^{(\mathrm{KF})}$
- Establish connections between the instanton parameter A and spinor density $\bar{C}C$
- Enable consistent generalization of the model to different dimensions

3.2 Four-Dimensional Special Case

For $D = 4$, the Keçeci-Fujii model takes the following simplified form:

$$\mathcal{L}_{\mathrm{KF}}^{(4)} = \frac{1}{2}(\partial_\mu \partial_\nu \varphi)^2 + \frac{1}{2}(\partial_\mu \bar{\psi})(i\partial\!\!\!/)(\partial_\mu \psi) + \sum_{j=0}^{4} \alpha_j^{(\mathrm{KF})} e^{(4-j)\beta\varphi/4} (\bar{\psi}\psi)^j. \tag{10}$$

In this case, the Keçeci recurrence relations yield the following solutions:

Table 1: Keçeci-Fujii model solutions for D=4 [4]

Case	$g = \boldsymbol{\alpha_2^{(\mathrm{KF})}}$	$\boldsymbol{A}$	$\boldsymbol{\bar{C}C}$
Pure solution	0	$-2, 2, 0$	$\mp 40/\beta^2, 0$
$g = \beta^2/32$	$\beta^2/32$	$-1/2$	-
$g = 3\beta^2/136$	$3\beta^2/136$	4	-
Special solution	1/4	Special	∓ 5

4 Keçeci-Fujii Model in Quantum Information Context

4.1 Connection with Topological Quantum Computing

The instanton solutions in the Keçeci-Fujii model show remarkable similarities with fundamental concepts of topological quantum computing:

1. **Conservation of Topological Charge**: The integer topological charge (Q) of instantons has a mathematical structure similar to the anyonic statistics of topological qubits.

2. **Quantum Tunneling Dynamics**: The instanton solutions in the model describe quantum tunneling between different topological sectors. This is analogous to the mechanism used in quantum annealing algorithms.

3. **Higher-Derivative Interactions**: The higher-derivative terms in the Keçeci-Fujii model may correspond to effective interactions in many-body quantum systems.

4.2 In the Context of Holographic Principle

The conformal structure of the model naturally connects with holographic duality:

$$\mathcal{Z}_{\text{Keçeci-Fujii}}[\varphi_0, \psi_0] = \mathcal{Z}_{\text{Gravity}}[\phi|_{\text{boundary}} = \varphi_0, \Psi|_{\text{boundary}} = \psi_0] \tag{11}$$

The higher-derivative terms in the Keçeci-Fujii model correspond to higher-derivative gravity theories on the dual gravitational side.

5 Future Research Proposals

5.1 Experimental Applications

1. **Quantum Simulation**: Quantum simulation of simplified versions of the Keçeci-Fujii model using cold atom systems or superconducting qubits.

2. **NISQ Algorithms**: Utilization of the model's instanton dynamics in optimization algorithms on near-term quantum processors.

3. **Topological Materials**: Experimental investigation of phase structures predicted by the model in topological insulators.

5.2 Theoretical Developments

1. **Quantum Correction of Keçeci-Fujii Model**: Calculation of complete quantum corrections and study of behavior under RG flows.

2. **Supersymmetric Extension**: Construction of an $\mathcal{N} = 2$ supersymmetric version of the Keçeci-Fujii model.

3. **Holographic Dual**: Establishment of a complete holographic dual theory for the model.

6 Conclusion and Assessment

In this study, we have presented the **Keçeci-Fujii model**, derived from the Akdeniz-Dane model and completed with Keçeci analysis on Fujii's generalization. The fundamental characteristics of the model are:

1. **Historical Continuity**: Akdeniz-Dane (1985) $\rightarrow$ Fujii (1989) $\rightarrow$ Keçeci (2011) $\rightarrow$ Keçeci-Fujii

2. **Mathematical Consistency**: Parameter space constraints provided by Keçeci recurrence relations

3. **Physical Depth**: Combination of instanton solutions, conformal symmetry, and higher-derivative interactions

4. **Modern Connections**: Natural connections with quantum computing, holography, and topological materials

The Keçeci-Fujii model offers a productive research platform that bridges classical conformal field theory with modern quantum technologies. The structures proposed by the model both shed light on fundamental questions in theoretical physics and provide a roadmap for experimental applications.

The analysis initiated in our previous work [[4], [5], [21], [22], [20]] has been extended and deepened in this work within the context of modern physics. Future studies should be directed toward investigating the experimentally testable predictions of the model and understanding its interaction with quantum information resources.

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Model	**Dimension**	**Derivative Level**	**Contribution**
Akdeniz-Dane (1985)	2D	First derivative	Basic scalar-spinor interaction
Fujii (1989)	2nD	Higher derivative	Dimension and derivative generalization
Keçeci (2011)	2nD	Higher derivative	Recurrence relations, coupling analysis
Keçeci-Fujii	2nD	Higher derivative	Modern applications, quantum connections

Table 2: Historical development process of the Keçeci-Fujii model

A Keçeci-Fujii Model: Mathematical Details

A.1 Developmental Process of the Model

A.2 Derivation of Keçeci Recurrence Relations

The recurrence relations we derived in our previous work [4] are obtained from the consistency equations (8) and (9) for the instanton-like solutions (6) and (7).

A.3 Symmetry Properties

The Keçeci-Fujii model preserves the following symmetries:

- Conformal symmetry (classical level)
- Global $U(1)$ symmetry: $\psi \to e^{i\theta}\psi$
- Discrete symmetry: $\varphi \to -\varphi$, $\beta \to -\beta$ (under certain conditions)

A.4 Quantum Information Parameters

Quantum information interpretation of Keçeci-Fujii model parameters:

Parameter	Physical Meaning	Quantum Information Correspondence
$\alpha_j^{(\mathrm{KF})}$	Coupling constants	Interaction strength
β	Liouville coupling	Scale invariance parameter
A	Instanton size	Tunneling amplitude
$\bar{C}C$	Spinor density	Entanglement measure
$D = 2n$	Spacetime dimension	Analog to Hilbert space dimension

Table 3: Quantum information interpretations of Keçeci-Fujii model parameters